New applications of dipole Monte Carlo implementations

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Introduction

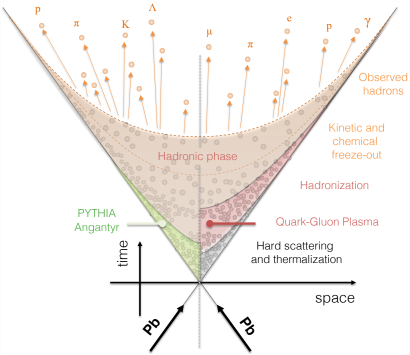
- Everywhere we look, we find heavy ion behavior!
- Monte Carlos for pp physics have had:
  1. No space–time structure.
  2. No heavy ion collisions.
  3. No collective effects.
- And that was a problem!

At least do enough for a non-QGP baseline.

But if it works, how far can we go?

The key differences between standard approaches

- Standard MC approach: Matrix element, parton shower + string hadronization.
- Note different time-scales.



(Figure: D. D. Chinellato)

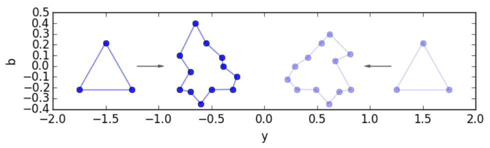
Outline

- Initial state geometry with Monte Carlo.
    1. From Mueller dipoles to event geometry.
    2. Fluctuating cross sections.
    3. Towards EIC.
- Matching to a multi-parton interactions.
    1. Pythia and the Angantyr model.
    2. Fluctuations in parton level geometry.
- From geometry to collectivity.
    1. The string shoving model.
    2. Shoving and Angantyr.
    3. Response to geometry.

Mueller dipole initial states

The aim and the means

A reasonable calculation of initial state geometry.
Fluctuating nucleon–nucleon cross sections.
MC implementation of Mueller dipoles.

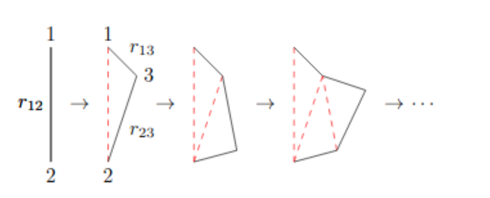


- Projectile and target cascades evolved for each event.
- Formalism in impact parameter and rapidity.
- Single-event spatial structure.

A step back, BFKL, B-JIMWLK and all that...

- Start with Mueller dipole branching probability:

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}y} = \mathrm{d}^2\vec{r}_3 \; \frac{N_c \alpha_s}{2\pi^2} \frac{r_{12}^2}{r_{13}^2 r_{23}^2} \equiv \mathrm{d}^2\vec{r}_3 \; \kappa_3.$$



- Evolve any observable $O(y) \rightarrow O(y + \mathrm{d}y)$ in rapidity:

$$\bar{O}(y+\mathrm{d}y) = \mathrm{d}y \int \mathrm{d}^2\vec{r}_3 \, \kappa_3 \left[O(r_{13}) \otimes O(r_{23})\right] + O(r_{12}) \left[1 - \mathrm{d}y \int \mathrm{d}^2\vec{r}_3 \; \kappa_3\right]$$

$$\rightarrow \frac{\partial \bar{O}}{\partial y} = \int \mathrm{d}^2\vec{r}_3 \; \kappa_3 \left[O(r_{13}) \otimes O(r_{23}) - O(r_{12})\right].$$

A powerful formalism!

- Example: S-matrix (eikonal approximation, b-space):

$$O(r_{13}) \otimes O(r_{23}) \to S(r_{13})S(r_{23})$$

- Change to $T \equiv 1 - S$:

$$\frac{\partial \overline{\langle T \rangle}}{\partial y} = \int \mathrm{d}^2 \vec{r}_3 \ \kappa_3 \left[\langle T_{13} \rangle + \langle T_{23} \rangle - \langle T_{12} \rangle - \langle T_{13} T_{23} \rangle \right].$$

- B-JIMWLK equation, but could be written with other observables.
- Example: Average dipole coordinate ($\langle z \rangle$):

$$\frac{\partial \overline{\langle z \rangle}}{\partial y} = \int \mathrm{d}^2 \vec{r}_3 \kappa_3 \left(\frac{1}{3} z_3 - \frac{1}{6}(z_1 + z_2) \right).$$

Monte Carlo implementation

Drawbacks to analytic approach

Involved observables are hard!
Not obvious how to include sub-leading effects.
Not obvious how to treat exclusive final states.

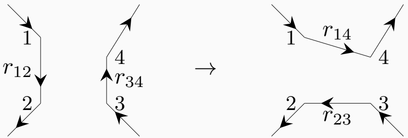
- The MC way is a tradeoff: formal precision vs. pragmatism.
- Get for free: Rest of the MC infrastructure.
- Practically a parton shower-like implementation.
- Step 1: Modify splitting kernel with Sudakov:

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}y\ \mathrm{d}^2\vec{r}_3} = \frac{N_c\alpha_s}{2\pi^2}\frac{r_{12}^2}{r_{13}^2 r_{23}^2}\exp\left(-\int_{y_{\min}}^{y}\mathrm{d}y\mathrm{d}^2\vec{r}_3\ \frac{N_c\alpha_s}{2\pi^2}\frac{r_{12}^2}{r_{13}^2 r_{23}^2}\right)$$

- Winner-takes-it-all algorithm generates emission up to maximal rapidity.
- Throws away the non-linear term in the cascade.

Colliding dipole chains & unitarity

- Have: Evolved dipole chain á la BFKL.
- Dipole cross section in large-N_c limit (consistency with evolution):



$$\frac{\mathrm{d}\sigma_{\mathrm{dip}}}{\mathrm{d}^2\vec{b}} = \frac{\alpha_s^2 C_F}{N_c} \log^2 \left[\frac{r_{13} r_{24}}{r_{14} r_{23}}\right]$$

$$\to \frac{\alpha_s^2}{2} \log^2 \left[\frac{r_{13} r_{24}}{r_{14} r_{23}}\right] \equiv f_{ij}$$

- Unitarized scattering amplitude: $T(\vec{b}) = 1 - \exp\left(-\sum_{ij} f_{ij}\right)$

Effects beyond leading log

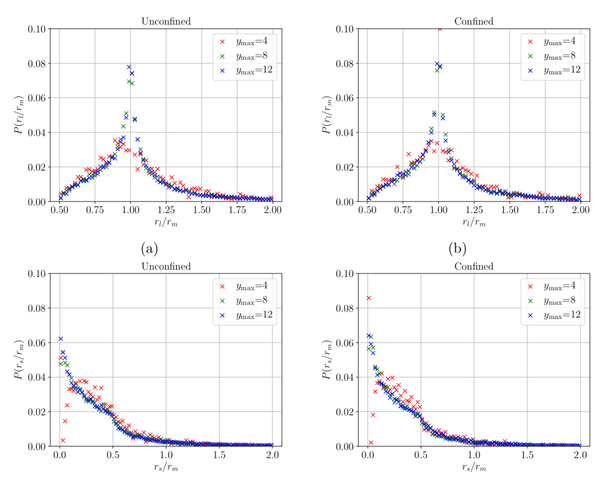
Some details

A dipole has a rapidity y, and a $p_\perp$ related to its size $p_\perp \; \hbar/r$. Thus its lightcone momenta is $p_\pm = p_\perp \exp(\pm y)$.

- Energy-momentum conservation from bounded p_- translate to upper bound on dipole sizes.
- Running α_s: Easily included per-splitting.
- Non-eikonal effects: recoil distributed on emitters in $p_+, p_\perp$, and thus also y.
- Confinement: Explicit confinement scale (or fictitious gluon mass) entering evolution and collision.
- Unitarized scattering amplitude resums $1/N_c^2$ terms in interaction, equivalent to multi-pomeron exchanges in interaction frame.

Example: confinement → hot-spots

- MC makes it easy to switch physics effects on and off.
- More activity around end-points: Hot-spots!
- Initial triangle by hand. Less important at high energies, but deserves more thought.



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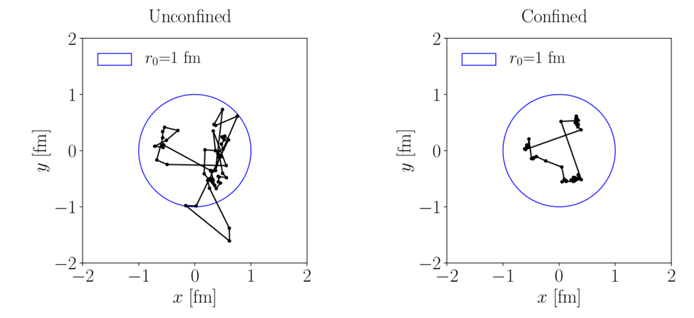


- Dynamically generated!

Good–Walker & cross sections

- Cross sections from $T(\vec{b})$ with normalizable particle wave functions:

$$\sigma_{\rm tot} = 2\int \mathrm{d}^2\vec{b}\Gamma(\vec{b}) = 2\int \mathrm{d}^2\vec{b}\ \langle T(\vec{b})\rangle_{p,t}$$

$$\sigma_{\rm el} = \int \mathrm{d}^2\vec{b}|\Gamma(\vec{b})|^2 = \int \mathrm{d}^2\vec{b}\ \langle T(\vec{b})\rangle_{p,t}^2$$

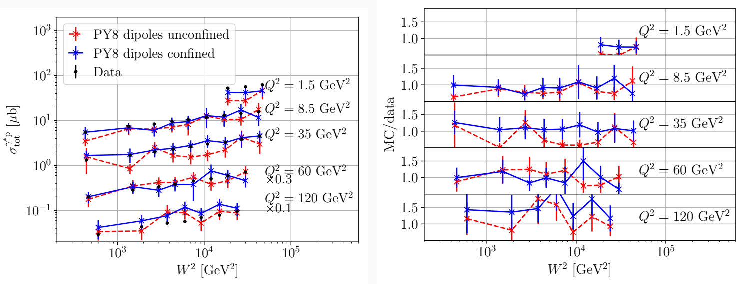
$$B_{\rm el} = \frac{\partial}{\partial t}\log\left(\frac{\mathrm{d}\sigma_{\rm el}}{\mathrm{d}t}\right)\Big|_{t=0} = \frac{\int \mathrm{d}^2\vec{b}\ b^2/2\ \langle T(\vec{b})\rangle_{p,t}}{\int \mathrm{d}^2\vec{b}\ \langle T(\vec{b})\rangle_{p,t}}$$

- Or with photon wave function:

$$\sigma^{\gamma^*\mathrm{p}}(s) = \int_0^1 \mathrm{d}z \int_0^{r_{\rm max}} r\mathrm{d}r \int_0^{2\pi} \mathrm{d}\phi \left(|\psi_L(z,r)|^2 + |\psi_T(z,r)|^2\right)\sigma_{\rm tot}(z,\vec{r})$$

Model parameters

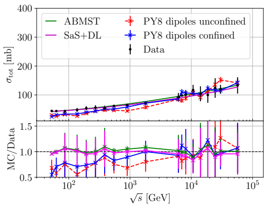
- This means that all parameters (4) can be tuned to cross sections

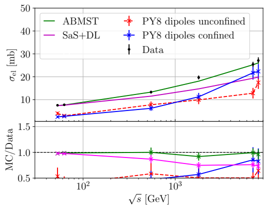


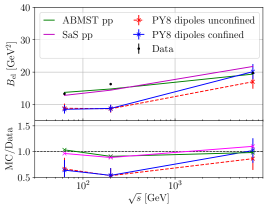
- Could constrain better in *ep* with eg. vector meson production.

Model parameters II

- Same parameters should describe pp, adds more data to the tuning.







- Not as good as dedicated (Regge-based) models.
- Accuracy not the point, control of physics features is!

Cross section colour fluctuations

- Cross section fluctuates event by event: important for pA, $\gamma^* A$ and less AA.
- Projectile remains frozen through the passage of the nucleus.
- Consider fixed state (k) projectile scattered on single target nucleon:

$$\Gamma_k(\vec{b}) = \langle \psi_S | \psi_I \rangle = \langle \psi_k, \psi_t | \hat{T}(\vec{b}) | \psi_k, \psi_t \rangle =$$

$$(c_k)^2 \sum_t |c_t|^2 T_{tk}(\vec{b}) \langle \psi_k, \psi_t | \psi_k, \psi_t \rangle =$$

$$(c_k)^2 \sum_t |c_t|^2 T_{tk}(\vec{b}) \equiv \langle T_{tk}(\vec{b}) \rangle_t$$

- And the relevant amplitude becomes $\langle T^{(nN_i)}_{t_i,k}(\vec{b}_{ni}) \rangle_t$

Fluctuating nucleon-nucleon cross sections

- Let nucleons collide with total cross section $2\langle T\rangle_{p,t}$
- Inserting frozen projectile recovers total cross section.
- Consider instead inelastic collisions only (color exchange, particle production):

$$\frac{\mathrm{d}\sigma_{\mathrm{inel}}}{\mathrm{d}^2\vec{b}} = 2\langle T(\vec{b})\rangle_{p,t} - \langle T(\vec{b})\rangle_{p,t}^2.$$

- Frozen projectile will not recover original expression, but requre target average first.

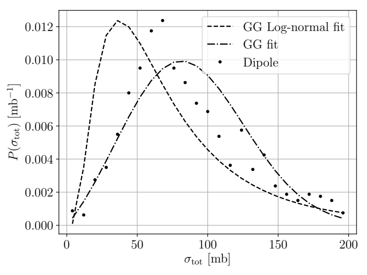
$$\frac{\mathrm{d}\sigma_{w}}{\mathrm{d}^2\vec{b}} = 2\langle T_k(\vec{b})\rangle_p - \langle T_k^2(\vec{b})\rangle_p = 2\langle T(\vec{b})\rangle_{t,p} - \langle\langle T(\vec{b})\rangle_t^2\rangle_p$$

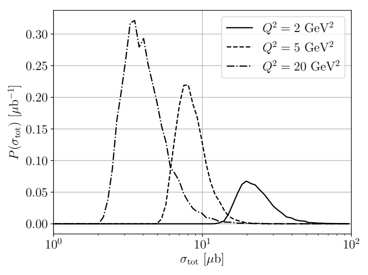
- Increases fluctuations! But pp can be parametrized.

EIC adds more complications

- For $\gamma^* A$ collisions the trick can be repeated.
- But photon wave function collapse to previous result at first hit.

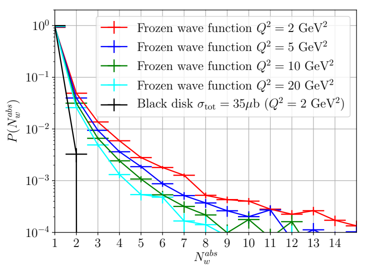
$$\frac{\mathrm{d}\sigma_w}{\mathrm{d}^2\vec{b}} = \int \mathrm{d}z \int \mathrm{d}^2\vec{r}\,(|\psi_L(z,\vec{r})|^2 + |\psi_T(z,\vec{r})|^2)(2\langle T(\vec{b})\rangle_{t,p} - \langle\langle T(\vec{b})\rangle_t^2\rangle_p).$$

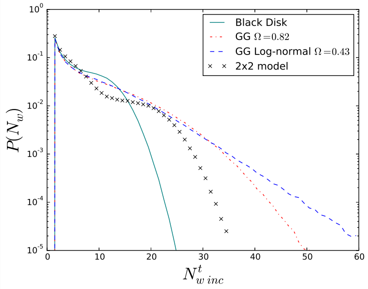




Drastic for number of wounded nucleons

- More multi-hit events, meaning more background.
- Clearly non-negligible, lesson already learned in p-Pb at LHC.





The story so far

- Mueller dipole MC for fluctuations and impact parameter space.
- Drastic consequenses for wounded nucleons.
- Must be coupled to particle production.
- ...and to initial spatial parton density.

Detour: MPIs in PYTHIA8 pp (Sjöstrand and Skands: arXiv:hep-ph/0402078)

- Several partons taken from the PDF.
- Hard subcollisions with $2 \to 2$ ME:

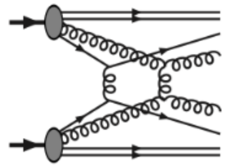


Figure T. Sjöstrand

$$\frac{d\sigma_{2\to 2}}{dp_\perp^2} \propto \frac{\alpha_s^2(p_\perp^2)}{p_\perp^4} \to \frac{\alpha_s^2(p_\perp^2 + p_{\perp 0}^2)}{(p_\perp^2 + p_{\perp 0}^2)^2}.$$

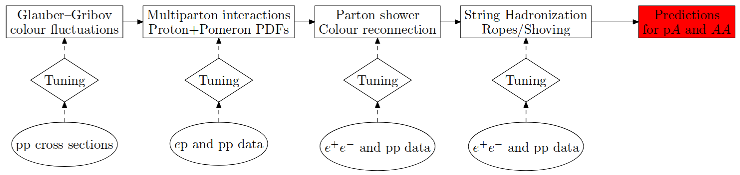
- Momentum conservation and PDF scaling.
- Ordered emissions: $p_{\perp 1} > p_{\perp 2} > p_{\perp 4} > \ldots$ from:

$$\mathcal{P}(p_\perp = p_{\perp i}) = \frac{1}{\sigma_{nd}} \frac{d\sigma_{2\to 2}}{dp_\perp} \exp\left[- \int_{p_\perp}^{p_{\perp i-1}} \frac{1}{\sigma_{nd}} \frac{d\sigma}{dp'_\perp} dp'_\perp \right]$$

- Picture blurred by CR, but holds in general.

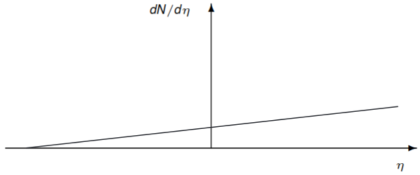
Angantyr – the Pythia heavy ion model (CB, G. Gustafson, L. Lönnblad: arXiv:1607.04434, += Shah: arXiv:1806.10820)

- Pythia MPI model extended to heavy ions since v. 8.235.
  1. Glauber geometry with Gribov colour fluctuations.
  2. Attention to diffractive excitation & forward production.
  3. Hadronize with Lund strings.



Particle production: Wounded nucleons

- Simple model by Białas and Czyz.
- Wounded nucleons contribute equally to multiplicity in η.
- Originally: Emission function $F(\eta)$ fitted to data.



$$\frac{dN}{d\eta} = F(\eta)$$ (single wounded nucleon

- Angantyr: No fitting to HI data, but include model for emission function.
- Model fitted to reproduce pp case, high $\sqrt{s}$, can be retuned down to 10 GeV.

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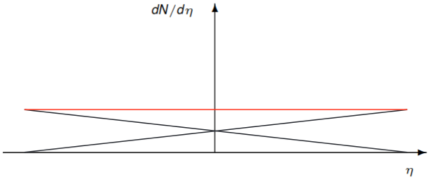

$$\frac{dN}{d\eta} = F(\eta) + F(-\eta) \qquad \text{(pp)}$$

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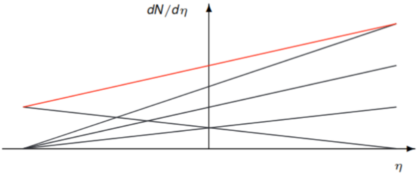

$$\frac{dN}{d\eta} = w_t F(\eta) + \quad F(-\eta) \qquad \text{(p}A\text{)}$$

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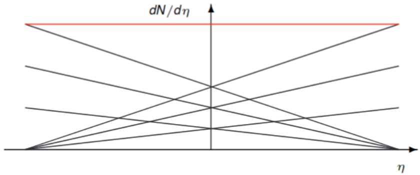

$$\frac{dN}{d\eta} = w_t F(\eta) + w_p F(-\eta) \qquad (AA)$$

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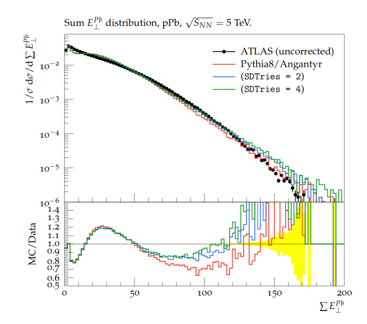
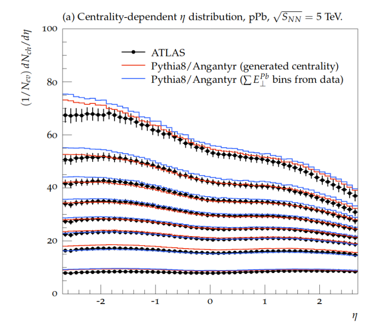
Some results - pPb

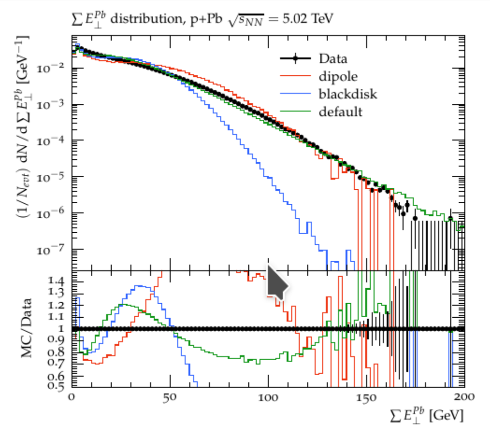
- Centrality measures are delicate, but well reproduced.
- So is charged multiplicity.





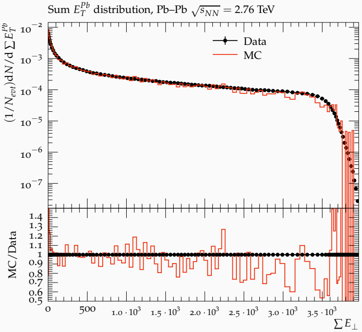
Direct dipole fluctuations

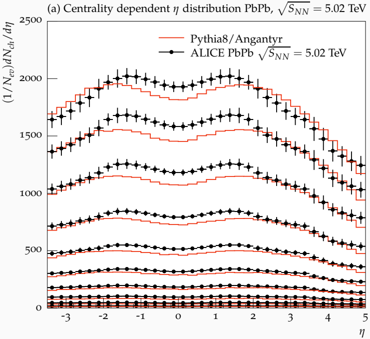
- Comparison between direct calculation (slow), and parametrized (fast).



Basic quantities in AA

- Reduces to normal Pythia in pp. In pA in AA:
    1. Good reproduction of centrality measure.
    2. Particle density at mid–rapidity.





- Geometric quantities from matching to dipole calculations coming up.

Parton vertices

- Parton vertices assigned according to dipole calculation.
- Cannot be done from first principles!

Priniciples of vertex assignments

- ◇ Dipole cascade branches go on-shell *iff* colliding with another.
- ◇ Partonic sub-collisions ordered in importance, *i.e.* contribution to cross section.
- ◇ Further emissions by parton shower smears and recoils with a Gaussian.

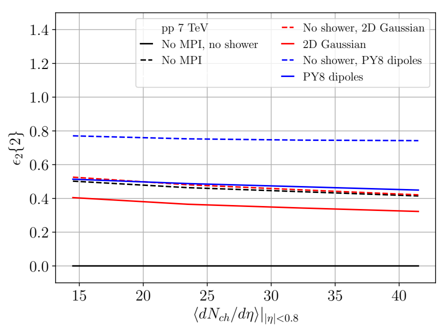
- Default model, for comparison, is proton mass distribution = 2D Gaussian.

Eccentricities

- Initial state anisotropy quantified:

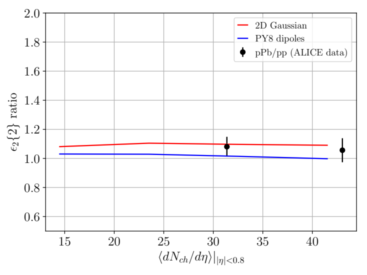
$$\epsilon_n = \frac{\sqrt{\langle r^2 \cos(n\phi)\rangle^2 + \langle r^2 \sin(n\phi)\rangle^2}}{\langle r^2 \rangle}.$$

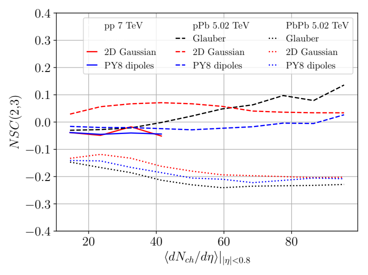
- ...and the usual higher moments.
- Beware infrared safety! $\rightarrow p_\perp/(p_\perp + p_{\perp,min})$



Could differences be measured?

- Differences visible, but p-Pb might be the best!

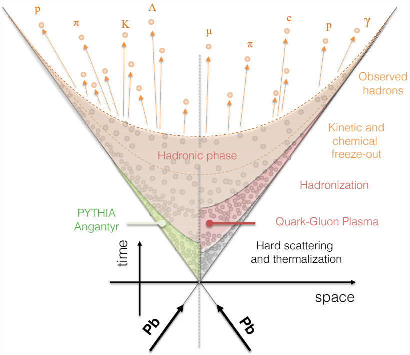




- NSC correlated flow coefficients, and scale out the magnitude.
- For p-Pb: Only negative in dipole picture.

Adding transport to final state

- The left side has now been established.
- Emphasis on colour fluctuations, forward production and partonic vertices.
- Rest of the talk: Transporting anisotropy to final state.



(Figure: D. D. Chinellato)

Microscopic final state collectivity

- Proposal: Model microscopic dynamics with interacting Lund strings
- Additional input fixed or inspired by lattice, few tunable parameters.

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$\tau \approx 0$ **fm:** Strings no transverse extension. No interactions, partons may propagate.

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$\tau \approx 2$ **fm:** Strings will hadronize. Possibly as a colour rope.

$\tau > 2$ **fm:** Possibility of hadronic rescatterings.

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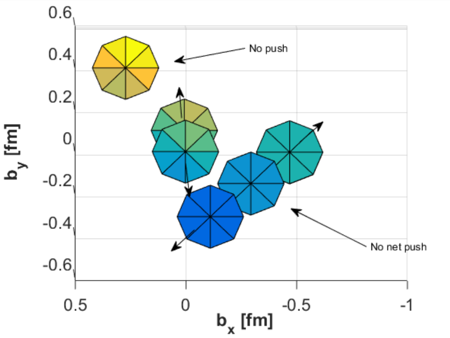
The cartoon picture

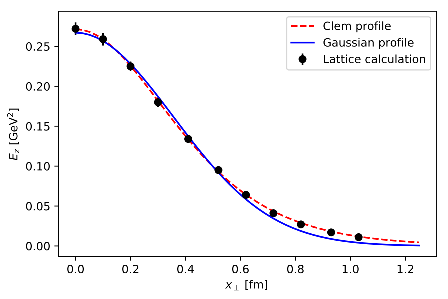
- Strings push each other in transverse space.
- Colour-electric fields → classical force.



👍 Transverse-space geometry.
👍 Particle production mechanism.
?? String radius and shoving force

MIT bag model, dual superconductor or lattice?

- Easier analytic approaches, eg. bag model: $\kappa = \pi R^2[(\Phi/\pi R^2)^2/2 + B]$
- Bad R 1.7 and dual sc. 0.95 respectively, shape of field is input.
- Lattice can provide shape, but uncertain R.



- Solution: Keep shape fixed, but R ballpark-free.

The shoving force

- Energy in field, in condensate and in magnetic flux.
- Let g determine fraction in field, and normalization N is given:

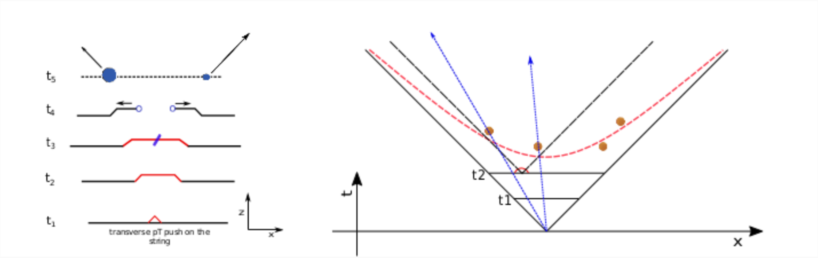
$$E = N \exp(-\rho^2/2R^2)$$

- Interaction energy calculated for transverse separation $d_\perp$, giving a force:

$$f(d_\perp) = \frac{g\kappa d_\perp}{R^2} \exp\left(-\frac{d_\perp^2}{4R^2}\right)$$

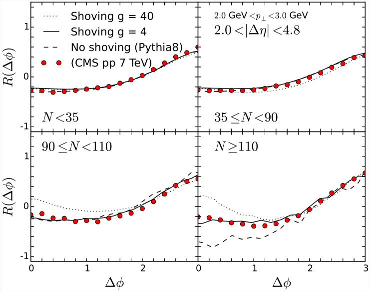
Monte Carlo details

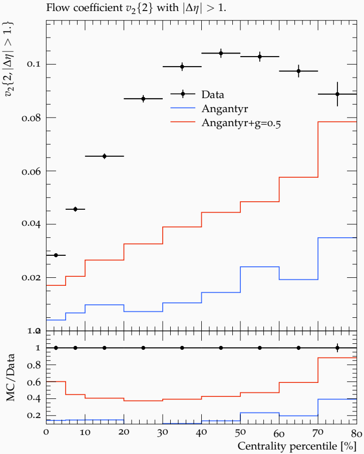
- Distance $d_\perp$ calculated in a frame where strings lie in parallel planes.
- Everything is two-string interactions.
- The shoving action implemented as a parton shower (again!)
- Push propagated along string, and distributed on final state hadrons.



Directly: varying results

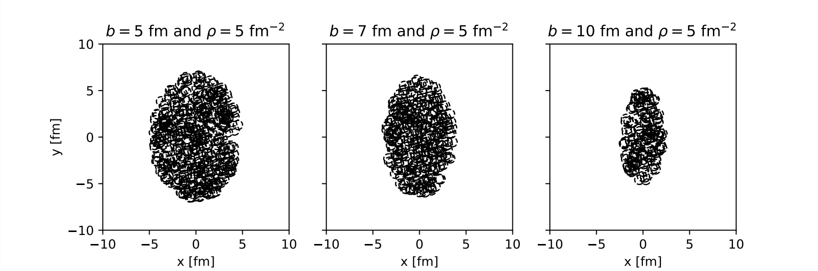
- Results are ok in pp, but off in AA.
- This is with the full initial state machinery = many things can go wrong.





Simpler toy initial state

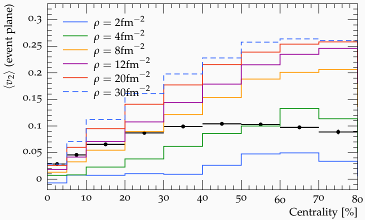
- Problematic: many soft gluons in final state.
- Corrections to string hadronization, saturation scale...
- Set up toy system of straight strings, study response to geometry.

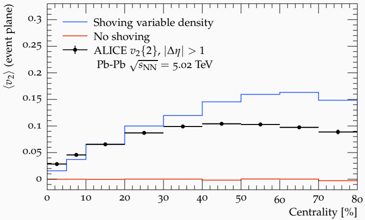


Fixed density close to reality

- Choose array of fixed densities, insert long strings, and calculate v_n:

$$v_n = \langle \cos(n(\phi - \Psi_n)) \rangle, \Psi_n = \frac{1}{n} \arctan \left(\frac{\langle p_\perp \sin(n\phi) \rangle}{\langle p_\perp \cos(n\phi) \rangle} \right)$$

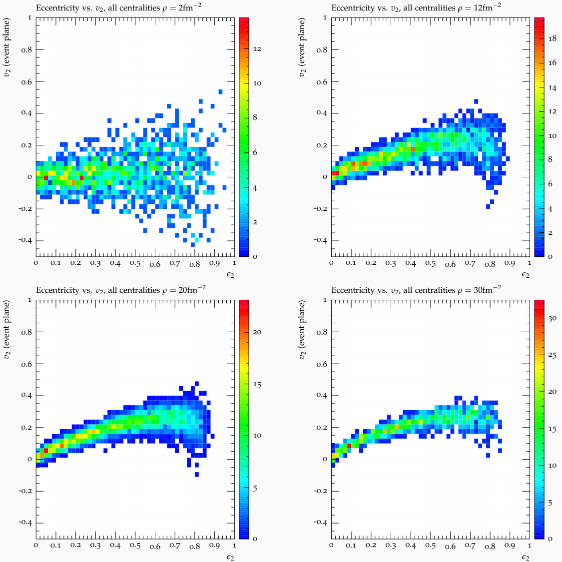




- Closer to data is nice, gives a path forward.

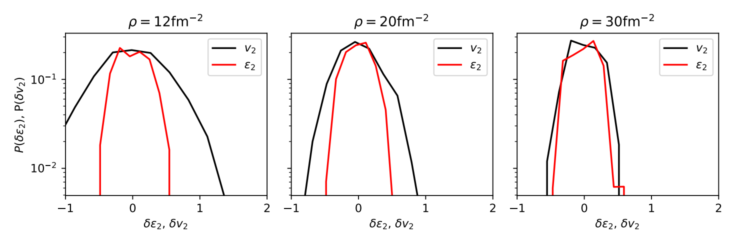
Scaling with initial eccentricity

Critical density? Critical for what?



Better: Rescaled variables

$$\delta\epsilon_2 = \frac{\epsilon_2 - \langle\epsilon_2\rangle}{\langle\epsilon_2\rangle} \text{ and } \delta v_2 = \frac{v_2 - \langle v_2\rangle}{\langle v_2\rangle}$$



- Scaling like hydro for large densities.
- ...but more fluctuations for low densities!

Summary: Dipoles and string interactions

- Mueller dipoles for geometry and IS fluctuations.
- Mapping to Pythia/Angantyr for particle production.
- Angantyr = p-A and AA final states, eA are coming.
- Huge opportinity: Control geometry and density at EIC.
- String shoving: interactions to generate transverse pressure.
- Interface to Angantyr still not perfect.
- Behaves like hydro in simple, high-density systems.

Thank you for the invitation!
Exciting times are still ahead!